



For All Practical Purposes

Mathematical Literacy in Today's World Tenth Edition

COMAP

**FOR
ALL PRACTICAL
PURPOSES**

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FOR ALL PRACTICAL PURPOSES

Mathematical Literacy
in Today's World

Tenth Edition

 W. H. FREEMAN
& COMPANY

A Macmillan Education Imprint

Publisher: Terri Ward
Acquisitions Editor: Nikki Miller
Marketing Manager: Cara LeClair
Development Editors: Leslie Lahr and Jorge Amaral
Executive Media Editor: Laura Judge
Assistant Media Editor: Danielle Lindsey
Associate Editor: Marie Dripchak
Editorial Assistant: Victoria Garvey
Marketing Assistant: Bailey James
Photo Editors: Cecilia Varas and Nick Ciani
Cover Designer: Blake Logan
Art Manager: Matthew McAdams
Text Designer: Marsha Cohen
Senior Project Editor: Vivien Weiss
Production Supervisor: Susan Wein
Illustrations: Eli Ensor and Lisa Winden
Cover Art: © Illustration source/Kiki Tikiriki
Composition: MPS Limited
Printing and Binding: RR Donnelley & Sons

Library of Congress Control Number: 2015939496

Student Edition (hardcover):

ISBN-13: 978-1-4641-2473-0

ISBN-10: 1-4641-2473-6

Student Edition (loose-leaf):

ISBN-13: 978-1-4641-2483-9

ISBN-10: 1-4641-2483-3

Instructor Comp Copy (hardcover):

ISBN-13: 978-1-4641-6633-4

ISBN-10: 1-4641-6633-1

NASTA-spec version:

ISBN-13: 978-1-4641-2480-8

ISBN-10: 1-4641-2480-9

© 2016 by COMAP, Inc.

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Printed in the United States of America

First printing

W. H. Freeman and Company

One New York Plaza Suite 4500

New York, NY 10004-1562

www.macmillanhighered.com

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- Sets
- Problem Solving
- Logic

- Geometry
- Counting and Probability
- Numeration Systems
- Personal Finance

Preface

To the Student

For All Practical Purposes, Tenth Edition, continues our effort to bring the excitement of contemporary mathematical thinking to the nonspecialist. In science and industry, mathematical models are the main tools for analyzing and solving problems that arise. In this book, our goal is to convey the power of mathematics by showing you the wide variety of problems that can be modeled and solved by quantitative means. An extensive package of supplements designed to make study time supremely effective complements the tenth edition. Highlights of the supplements package include the *Student Study Guide* and *Student Solutions Manual*. Between the text and the available resources, *For All Practical Purposes* offers you the tools to succeed in the course and apply your new knowledge to daily life experiences.

There are many ways to talk about why mathematics and its applications matter. You will hear expressions such as “mathematical literacy” or “quantitative literacy.” They mean, essentially, that math is important. It is important because knowing it can make your life easier. In other words, it can help explain how your world works. We created this course and this book because we know that not everyone looks at mathematics in this way.

In school, you spent a great deal of time learning the tools of mathematics—how to manipulate symbols and how to solve equations. In this course, you will spend time learning about the power of mathematics, which helps us understand many different parts of our everyday lives and the world itself. We hope this exploration will give you a broader sense of the subject and why we wanted you to take a math course every year you were in school. It’s “for all practical purposes” because, in a sense, you’ve learned to hammer nails and saw wood—and now we’re going to build houses.

Enjoy!

To the Instructor

Because *For All Practical Purposes* stresses the connections between contemporary mathematics and modern society, our text must be flexible enough to accommodate new ideas in mathematics and their new applications to our daily lives. We maintain this flexibility in the tenth edition.

Our primary goal for this edition was to further improve the ease of use for instructors and students alike. An extensive supplements package is available within LaunchPad, W. H. Freeman’s new online homework system. LaunchPad offers content that has been curated and organized for easy assignability in a simple but powerful interface. Assets integrated into LaunchPad include an interactive e-Book, LearningCurve, practice quizzes, exercise solutions, interactive applets, flashcards, video clips, and much more.

New to the Tenth Edition

New Algebra Review Appendix

An all-new Algebra Review Appendix offers reviews on basic concepts used in the text and includes straightforward examples, technology tips, and practice exercises. Answers to all practice exercises are also included. References to relevant sections of the Algebra Review Appendix appear throughout the text in the margins.

New Self Check Exercises

Self Check exercises have been added throughout the text, with answers at the end of each chapter, allowing students to check their understanding of new concepts as the material is being taught. Instructors can also use these exercises in class as part of their lectures.

New Examples

New and updated examples are included throughout the text to address new topics and changes to the material. Examples provide new topics for class discussion and new ways of relating to essential concepts.

New Exercises

- Exercise sets, including Skills Check questions, have been updated and refreshed.
- Over 200 Self Check exercises have been added throughout the chapters.
- The new Algebra Review Appendix includes over 200 practice exercises.
- New Chapter Review exercises have been added to the exercise sets. These exercises are not organized by section and test student understanding of the chapter material as a whole.

Part-Specific Content Changes

Part I: Management Science

- New discussion of the importance of operations research for improving health care (Chapters 1, 3, and 4).
- Revised example illustrating how to cut the costs of installing a local cable TV network (Chapter 2).
- A new case study, which closes Part I, offers a discussion of the ideas behind Alvin Roth and Lloyd Shapley's Nobel Prize-winning work about stable allocations, such as pairing hospitals and medical school graduates for residencies and pairing schoolchildren with schools they want to attend (school choice).

Part II: Statistics: The Science of Data

- Throughout Part II, more graphic displays have been added to help students visualize the material.
- Spotlight features have been added about the use of technology (TI-84 graphing calculator, Excel) for calculations, creation of graphic displays, and selection of simple random samples (Chapters 5–8).
- Expanded directions for constructing histograms and stemplots, along with examples on changing histogram class intervals and expanding stemplot stems (Chapter 5).

- Addition of an example showing computation of correlation from a formula (Chapter 6, Example 7).
- Explanation of least-squares criterion (Chapter 6).
- Addition of an example showing the dangers of extrapolation (Chapter 6, Example 10).
- Discussion of how the Pew Research Center modifies its telephone survey plans to correct for undercoverage (Spotlight 7.2).
- Expanded instructions on selecting a simple random sample (Chapter 7).
- Expanded discussion of ethics in experiments (Spotlight 7.3).
- Expanded discussion of the basic rules of probability (Chapter 8).
- Inclusion of estimation of probabilities based on survey data (Chapter 8).
- Coverage of conditional probability and Bayes' Rule (Section 8.2).

Part III: Voting and Social Choice

- Simplified coverage of the Banzhaf Power Index (Chapter 11).
- New section on the use of apportionment in presidential primaries, including an example with real data from the 2008 Democratic primary (Section 12.1).
- Improved presentation of spatial modeling by introducing discrete models first and using them to analyze the Electoral College (Chapter 12).

Part IV: Fairness and Game Theory

- Expanded coverage of the history of the apportionment of seats in the U.S. House of Representatives (Chapter 14).
- New section highlights mechanism design—designing a game to achieve a particular outcome (Section 15.4).
- New examples model a work location versus schedule decision, as well as the goalie/kicker interaction in a soccer penalty kick (Chapter 15).

Part V: The Digital Revolution

- New examples of check-digit schemes (Chapter 16).
- Expanded summary of error-detection schemes (Chapter 16).
- Expanded spotlight on history of bar codes (Chapter 16).

- New title for Chapter 17 to more accurately describe the content.
- New spotlight features, covering facts about social security numbers, enigma machines, Mavis Batey, Alan Turing, and smart cards (Chapter 17).
- Added coverage of permutation, Playfair, and Jefferson wheel ciphers (Chapter 17).
- New website and video suggestions added (Chapter 17).

Part VI: On Size and Growth

- Revised exercises that consider production of solar energy, consumption by electric cars, and units for measuring water use (Chapter 18).
- New spotlight featuring an award-winning young mathematician whose interest was prompted by rhythms that he learned from playing drums and reading Sanskrit poetry (Spotlight 19.2).
- New spotlight on the use of Fibonacci numbers in optimizing the design of a solar power plant (Spotlight 19.3).
- New examples of patterns on ancient and modern pottery (Chapter 19).
- New illustration of a Penrose pattern in San Francisco architecture, and new figures and exercises about the inflation of patterns, as well as a discussion of the Nobel Prize awarded for the discovery of quasicrystals (Chapter 20).
- New spotlight on mathematics and autism (Spotlight 20.5).

Part VII: Your Money and Resources

- Incorporation of current student loan interest rates into examples (Chapter 22).
- New section on discounted and add-on loans, with exercises about payday loans (Chapter 22).
- Example showing details of costs at a real estate closing (Chapter 22).
- New spotlight on how minimum-size catch limits manipulate fish genetics (Spotlight 23.4).
- New exercises on radioactive isotopes, including those used in medicine and those released in the meltdown of Japanese reactors (Chapter 23).

Focus on Accuracy

For this edition, we once again implemented a detailed accuracy-checking plan to sustain the quality of the exercises and solutions. To this end, we are very

grateful to Dennis Evans of Concordia University–Wisconsin and Paul McCombs of Rock Valley College.

Custom Options

In addition to the extensive topics covered in the text, more traditional chapters (including Problem Solving, Sets, Logic, Geometry, Counting and Probability, Numeration Systems, and Personal Finance) are available with *For All Practical Purposes* through custom publishing. For more information, please contact your W. H. Freeman representative or go to www.macmillanhighered.com/fapp10e. Restrictions apply.

Media and Supplements

The media and supplements package for the tenth edition has been updated to reflect changes in the book. Both instructors and students will benefit from the innovative materials available to them.



W. H. Freeman's new online homework system, **LaunchPad**, offers quality content that has been curated and organized for easy assignability in a simple but powerful interface. We have taken what we have learned from thousands of instructors and hundreds of thousands of students to create a new generation of W. H. Freeman/Macmillan technology.

Curated units. Combining a curated collection of videos, homework sets, tutorials, applets, and e-Book content, LaunchPad's interactive units give instructors a building block to use as is or as a starting point for customized learning units. A majority of exercises from the text can be assigned as online homework, including an abundance of algorithmic exercises. An entire unit's worth of work can be assigned in seconds, drastically reducing the amount of time it takes for instructors to have their course up and running.


Easily customizable. Instructors can customize the LaunchPad units by adding quizzes and other activities from our vast wealth of resources. They can also add a discussion board, a drop box, and an RSS feed, with just a few clicks. LaunchPad allows instructors to customize students' experiences as much or as little as desired.


Useful analytics. The gradebook quickly and easily allows instructors to look up performance metrics for classes, individual students, and individual assignments.

Intuitive interface and design. The student experience is simplified. Students' navigation options and expectations are clearly laid out at all times, ensuring they can never get lost in the system.


Assets integrated into LaunchPad include the following:

Interactive e-Book. Every LaunchPad e-Book comes with powerful study tools for students, video and multimedia content, and easy customization for instructors. Students can search, highlight, and bookmark, making it easier to study and access key content. And teachers can ensure that their classes get just the book they want to deliver by customizing and rearranging chapters; adding and sharing notes and discussions; and linking to quizzes, activities, and other resources.

 **LearningCurve** provides students and instructors with powerful adaptive quizzing, a game-like format, direct links to the e-Book, and instant feedback. The quizzing system features questions tailored specifically to the text, and it adapts to students' responses, providing material at different difficulty levels and topics based on student performance.

 **SolutionMaster** offers an easy-to-use web-based version of the instructor's solutions, allowing instructors to generate a solution file for any set of homework exercises.

Other online homework options include:

 **WebAssign** integrates the text exercises from *For All Practical Purposes*, Tenth Edition, into a popular and trusted online homework system, making it easy to assign algorithmically generated homework and quizzes.

Student Resources

- **Student Solutions Manual** provides solutions to the odd-numbered exercises, with step-by-step solutions to select problems.
- **Student Study Guide** offers study tips and tools to help students gain a better understanding of course material, including key ideas for each section and additional examples and practice exercises.
- **Interactive applets** help students master key mathematical concepts and work exercises.
- **Math Clips** are animated whiteboard videos that illuminate key concepts in the text by showing students step-by-step solutions to selected exercises.
- **Self-quizzes, flash cards,** and other projects offer additional study help.

Instructor Resources

- **Instructor's Guide with Full Solutions** includes teaching suggestions, chapter comments, and detailed solutions to all exercises.
- **Teaching Guide for First-Time Instructors** helps instructors, adjuncts, and teaching assistants plan their course more easily and effectively. This guide also offers fresh perspectives and ideas to experienced instructors.
- **Test Bank** offers thousands of multiple-choice questions.
- **Lecture slides** offer a detailed lecture presentation of concepts covered in each chapter of *For All Practical Purposes*, Tenth Edition.
- **Clicker Questions** are available for each chapter.

Companion Website

www.macmillanhighered.com/fapp10e This open-access website provides students with access to the applets referenced throughout the text.

Acknowledgments

For All Practical Purposes continues to evolve in great part because of our many friends and colleagues who have offered suggestions, comments, and corrections. We are grateful to them all.

Michael Allen, *Glendale Community College*
Hamid Attarzadeh, *Kentucky Community and Technical College System*
Robert J. Bass, *Gardner-Webb University*
Debra D. Bryant, *Tennessee Technological University*
Annette M. Burden, *Youngstown State University*
Christine Cedzo, *Gannon University*
Gina Poore Dunn, *Lander University*
Dennis Evans, *Concordia University*
Kevin Ferland, *Bloomsburg University*
Gregory Goeckel, *Presbyterian College*
Doug Greiner, *Rogers State University*
Samuel S. Gross, *Bloomsburg University*
Patricia Humphrey, *Georgia Southern University*
Jamie L. King, *Arkansas Tech University*
Jay Malmstrom, *Oklahoma City Community College*
Paul McCombs, *Rock Valley College*
Linda McGuire, *Muhlenberg College*
Amanda Nutt, *Western Kentucky University*
Jacquelyn O'Donohoe, *Plymouth State University*
Daniel Pinzon, *Georgia Gwinnett College*
Stacy Reagan, *Caldwell Community College*
David Shannon, *Transylvania University*
Sharon Sullivan, *Catawba College*
Susan Toma, *Madonna University*
Kim Y. Ward, *Eastern Connecticut State University*
Weicheng Xuan, *Arizona Western College*

We owe our appreciation to the people at W. H. Freeman and Company who participated in the development and production of this edition. We

wish especially to thank the editorial staff for their tireless efforts and support. Among them are Terri Ward, Publisher; Leslie Lahr and Jorge Amaral, Development Editors; Laura Judge, Executive Media Editor; Marie Dripchak, Associate Editor; Victoria Garvey, Editorial Assistant; Vivien Weiss, Senior Project Editor; Susan Wein, Production Supervisor; Matthew McAdams, Art Manager; Blake Logan, Cover Designer; Cecilia Varas and Nick Ciani, Photo Editors; Jennifer MacMillan, Felicia Ruocco, and Hilary Newman, Permissions; Patti Brecht and Laura Cooney, copyeditors; Edward Dionne, Project Manager, MPS Limited.

We'd also like to give special thanks to the authors of the ancillaries to accompany this tenth edition: Lauren Fern, University of Montana, Instructor's Guide with Full Solutions and Student Solutions Manual; Doug Greiner, Rogers State University, Teaching Guide for First-Time Instructors and Student Study Guide; Hee Seok Nam, Washburn University, Practice Quizzes and Test Bank; Kathy Rodgers, University of Southern Indiana, Clicker Questions and Lecture Slides.

Through ten editions, this text has been used by well over a million students. When we first suggested our new approach, we were turned down by every major (and minor) textbook publisher. Only W. H. Freeman, under the leadership of Linda Chaput and the faith of mathematics editor Jerry Lyons, was willing to take a chance. That chance has permanently changed the face of introductory undergraduate mathematics. Words cannot express the gratitude we feel for the staffs of W. H. Freeman and COMAP and for the authors past and present through these almost 30 years. To them and everyone who made our purposes practical, we offer our appreciation for an exciting and exhilarating ride.

Solomon Garfunkel, COMAP

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**FOR
ALL PRACTICAL
PURPOSES**

Part I



Management Science

Chapter 1

Urban Services

Chapter 2

Business Efficiency

Chapter 3

Planning and Scheduling

Chapter 4

Linear Programming and the Transportation Problem

Getting through a typical day can be a challenge: getting to or from school or your job on time; finding a parking spot when you are late for a date; making sure you have food in the refrigerator or getting to your favorite fast-food restaurant to stay properly fed and “fueled” with coffee; making sure your body is fit by getting to the gym or exercising at home; and seeing the doctor for a regular checkup or when you are ill. While your personal life may seem complex, consider what goes on at any of the large, modern medical centers found across America, in cities and on university campuses. In a typical 24-hour period, babies will be born; people will die; a friend may need an emergency appendectomy; and a relative may need treatment for a heart attack, asthma attack, stroke, or puncture wound. And there are the more mundane things: blood tests, mammograms, garden variety X-rays, CT and MRI scans, elective surgery to remove a cyst, or a routine checkup. Medical centers need many kinds of workers to make them hum: doctors and nurses with different specialties, technicians of different kinds, workers who pay attention to the “business” aspects of the medical center—not to mention people who prepare food for the patients, clean the premises, or plow the access roads after a snowstorm.

So what does this have to do with mathematics? To deal with the emergencies and day-in and day-out demands, there must be specialists (doctors, nurses, technicians, etc.) and other workers either physically present or available by phone. The part of mathematics concerned with efficient operations of businesses and governments is called operations research (OR) or management science. The domain of OR includes resource allocation, scheduling, queues (waiting lines), inventory analysis, routing problems, and cost minimization, to mention but a few of OR’s growing areas of applicability. Medical centers rely on the expertise of mathematical specialists to make their operations run smoothly day and night, rain or shine, winter or summer, so that when you or your loved ones need to use the center, its services are there for you.

Chalk up another triumph for OR! What follows will help you, too, to know about and use such tools.

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Urban Services

1



Mitchell Funk/Getty Images

- 1.1 Euler Circuits
- 1.2 Finding Euler Circuits
- 1.3 Beyond Euler Circuits
- 1.4 Urban Graph Traversal Problems

The underlying theme of management science, also called **operations research (OR)**, is finding the best method for solving some problem—what mathematicians call the **optimal solution**. In some cases, the goal may be to finish a job or get somewhere as quickly as possible. In other situations, the objective might be to maximize profit or minimize cost. In this chapter, our goal is to save time (and usually taxpayer money) in traversing a street network while providing services such as checking parking meters, collecting garbage or bottles for recycling, de-icing roads, inspecting for potholes or gas leaks, or delivering packages or mail.

Let's begin by assisting the parking department of a city government. Most cities and many small towns have parking meters that must be regularly checked for parking violations or emptied of coins. We will use an imaginary town to show how management science techniques can help to make parking control more efficient.

1.1 Euler Circuits

The street map in Figure 1.1 is typical of many villages and cities across the United States, with streets, residential blocks, and a town park. Our job, or that of the commissioner of parking, is to find the most efficient route for the parking-control officer, who travels on foot, to check the meters in an area. Efficient routes save money. Our map shows only a small area, allowing us to start with an easy problem. But the problem occurs on a larger scale in all cities and towns and for larger areas. The bigger the region involved, the greater the potential for cost savings.

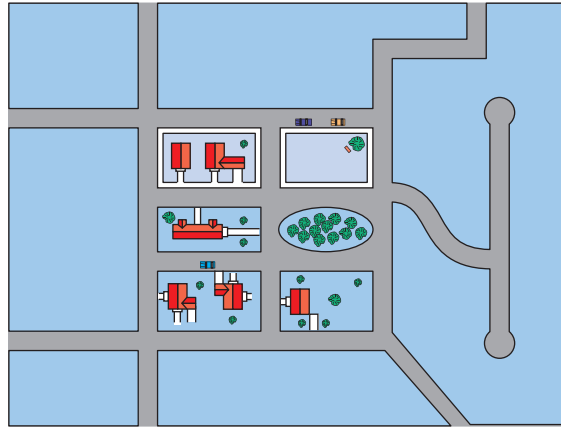


Figure 1.1 A street map for part of a town.

The commissioner has two goals in mind: (1) The parking-control officer must cover all the sidewalks that have parking meters without retracing any more steps than are necessary; and (2) the route should start and end at the same location, perhaps where the officer's vehicle is parked. To be specific, suppose there are only two blocks that have parking meters, the two lightly shaded blocks that are side by side toward the top of Figure 1.1. Suppose further that the parking-control officer must start and end at the upper left corner of the left-hand block. You might enjoy working out some routes by trial and error and evaluating their good and bad features. We are going to leave this problem for the moment and establish some concepts that will give us a better method than trial and error to deal with this kind of problem.

What Is a Graph?

DEFINITION

A **graph** is a finite set of dots and connecting curved or straight links. The dots are called **vertices** (a single dot is called a **vertex**), and the links are called **edges**. Each edge must connect two different vertices. A graph can represent a city map, a social network, a system of air routes, or electrical power lines.

Path and Circuit

DEFINITION

A **path** is a connected sequence of edges showing a route on the graph that starts at a vertex and ends at a vertex; a path is usually described by naming in turn the vertices visited in traversing it. A path that starts and ends at the same vertex is called a **circuit**.

EXAMPLE 1  **Parts of a Graph**

We can use the graph in Figure 1.2 to help explain these technical terms. The graph shown has five vertices and eight edges. The vertices represent cities, and the edges represent nonstop airline routes between them. We see that there is a nonstop flight between Berlin and Rome, but no such flight between New York and Berlin. There are several paths that describe how a person might travel with this airline from New York to Rome. The path that seems most direct is New York, London, Rome. But New York, Miami, Rome is also a path with only one “stop.” Furthermore, New York, London, Berlin, Rome is a path. We can describe these three paths as NLR , NMR , $NLBR$.

Another path would be New York, Miami, London, Berlin, Rome, which can be written $NMLBR$. An example of a circuit is Miami, Rome, London, Miami. It is a circuit because the path starts and ends at the same vertex. This circuit can best be described in symbols by $MRLM$. Another example of a circuit in this graph would be $LRBL$, which is the circuit involving the cities London, Rome, Berlin, and back to London. In this chapter, we are especially interested in circuits, just as we are in real life. Most of us end our day in the same place that we start it—at home!

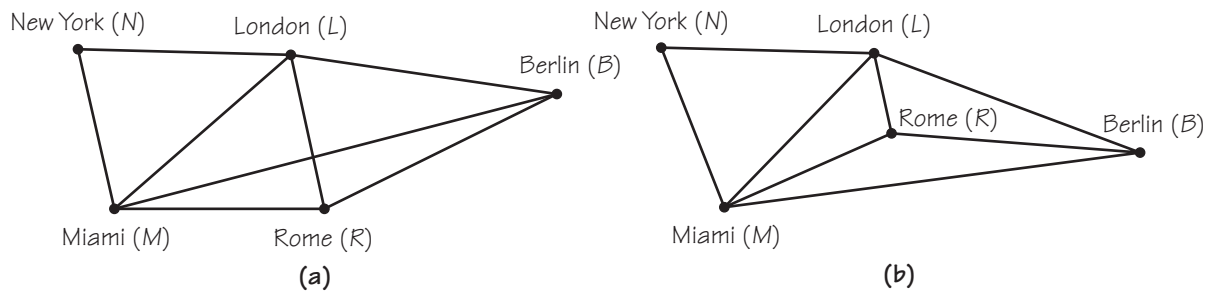
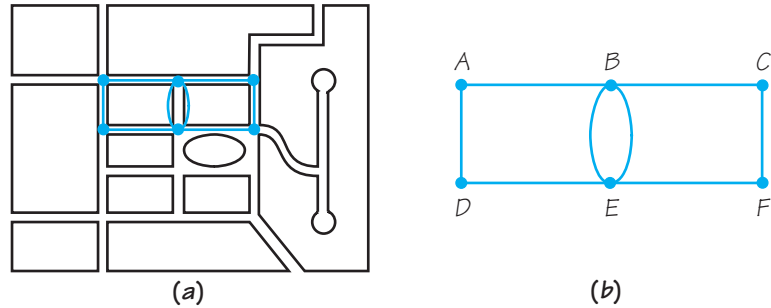


Figure 1.2 (a) The edges of the graph show nonstop routes that an airline might offer. (b) The graph in (a) redrawn without the accidental crossing.

Notice that the edges MB (which could also be denoted BM) and RL shown in Figure 1.2a meet at a point that has no label. Furthermore, this point does not have a dark dot. This is because this point does not represent a vertex of our graph; it does not represent a city. It arises as an “accidental” consequence of the way this diagram has been drawn. We could join M and B with a curved line segment so that the edges LR and MB do not cross, or redraw the diagram so as to avoid a crossing in this case. We will be working often in situations where graphs can be drawn without accidental crossings, and we will try to avoid such crossings when it is convenient to do so. However, there are infinitely many graphs for which—when they are drawn on a flat piece of paper—accidental crossings are unavoidable. (Figure 2.12 on page 52 is an example of such a graph.)

Returning to the case of parking enforcement in Figure 1.1, we can use a graph to represent the whole territory to be patrolled: Think of each street intersection as a vertex and each sidewalk that contains meters as an edge, as in Figure 1.3. Notice in Figure 1.3b that the width of the street separating the blocks is not explicitly represented; it has been shrunk to nothing. In effect, we are simplifying our problem by ignoring any distance traveled in crossing streets. In drawing graph diagrams such as those in Figure 1.3 or Figure 1.5, we usually use straight line segments to

Figure 1.3 (a) A graph superimposed upon a street map. The edges show which sidewalks have parking meters. (b) The same graph enlarged.



represent edges. However, sometimes we cannot avoid the use of “curves,” or we may prefer to use curved edges because they convey aspects of the original problem that we desire to emphasize.

The sequence of numbered edges in Figure 1.4a shows one circuit that covers all the meters. (Note that it is a circuit because its path returns to its starting point.) However, one edge is traversed three times. Figure 1.4b shows another solution that is better because its circuit covers every edge (sidewalk) exactly once. In Figure 1.4b, no edge is covered more than once, or *deadheaded* (a term borrowed from shipping, which means making a return trip without a load). When deadheading is required in an applied situation, such as inspecting parking meters or pothole inspection, typically time and effort is being spent but no productive work is accomplished because the productive work was done the first time the edge was covered (traversed).

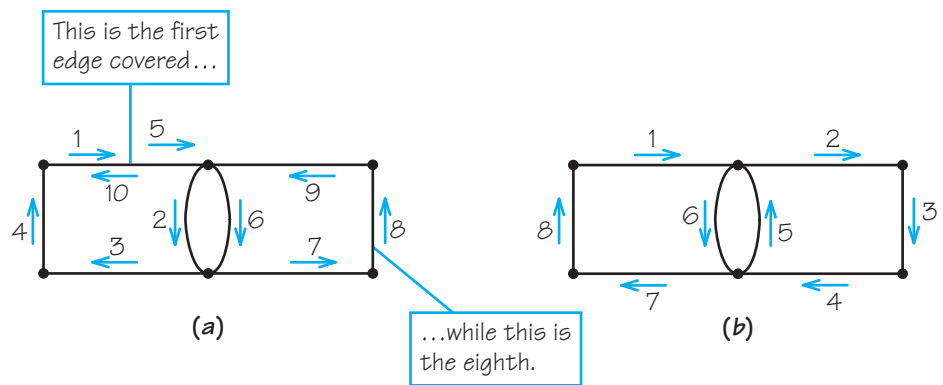


Figure 1.4 (a) A circuit and (b) an Euler circuit.

Euler Circuit

DEFINITION

A circuit that covers each edge of a graph once, but not more than once, is called an **Euler circuit**.

Figure 1.4b shows an Euler circuit. These circuits get their name from the great Swiss-born mathematician Leonhard Euler (pronounced *oy' lur*), who first studied them (see Spotlight 1.1). Euler was the founder of the theory of graphs, or graph theory. One of his first discoveries was that some graphs have no Euler circuits at all.

Leonhard Euler

SPOTLIGHT 1.1

Leonhard Euler (1707–1783) was remarkable in many ways. He was extremely prolific, publishing over 500 works in his lifetime. But he wasn't devoted just to mathematics; he was a people person, too. He was extremely fond of children and had 13 of his own, of whom only five survived childhood. It is said that he often wrote difficult mathematical works with a child or two in his lap.

Human interest stories about Euler have been handed down through three centuries. He was a prodigy at doing complex mathematical calculations under less than ideal conditions, and he continued to do them even after he became totally blind later in life. His blindness diminished neither the quantity nor the quality of his output. Throughout his life, he was able to mentally calculate in a short time what would have taken ordinary mathematicians hours of pencil-and-paper work. A contemporary claimed that Euler could calculate effortlessly, "just as men breathe, as eagles sustain themselves in the air." His collected works and numerous letters

to other scholars of his day are still being published.

Euler invented the idea of a graph in 1736 when he solved a problem in "recreational mathematics." He showed that it was impossible to stroll a route visiting the seven bridges of the German town of Königsberg exactly once. Ironically, in 1752 he discovered that three-dimensional polyhedra obey the remarkable formula $V - E + F = 2$ (that is, number of vertices - number of edges + number of faces = 2) but failed to give a proof because he did not analyze the situation using graph theory methods. Sometimes even a genius can miss something.



Jamie Abecasis/image/age fotostock

Leonhard Euler

For example, in the graph in Figure 1.5b, it would be impossible to start at one point, return to that starting vertex and cover all the edges without retracing some steps: If we try to start a circuit at the leftmost vertex, we discover that once we have left the vertex, we have "used up" the only edge meeting it. We have no way to return to our starting point except to reuse that edge. But this is not allowed in an Euler circuit. If we try to start a circuit at one of the other two vertices, we likewise can't complete it to form an Euler circuit.

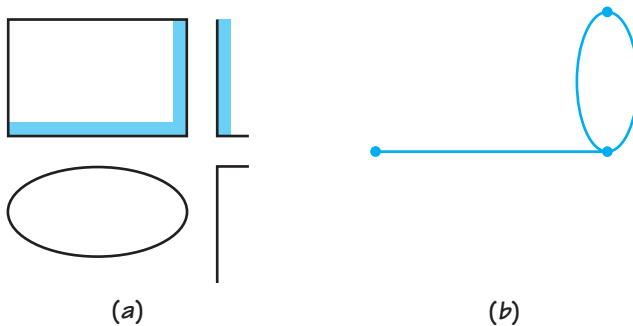


Figure 1.5 (a) The three shaded sidewalks cannot be covered by an Euler circuit. (b) The graph of the shaded sidewalks in part (a).

As mentioned in Spotlight 1.2, realistic problems of this type involve larger neighborhoods that might require the use of a computer. In addition, there may be other complications that might take us beyond the simple mathematics we want to stick to.